Error Propagation in Arithmetic Calculations

courtesy of http://www.nuclear.utah.edu/

Type of Calculation	Example*	Standard Deviation of x
Addition or Subtraction	$x = p + q - r$	$s_x = \sqrt{s_p^2 + s_q^2 + s_r^2}$
Multiplication or Division	$x = \frac{p \cdot q}{r}$	$s_x = x\sqrt{\left(\frac{s_p}{p}\right)^2 + \left(\frac{s_q}{q}\right)^2 + \left(\frac{s_r}{r}\right)^2}$
Exponentiation	$x = p^y$	$s_x = y \cdot x \frac{s_p}{p}$
Logarithm	$x = \log_{10} p$	$s_x = 0.434 \frac{s_p}{p}$
Natural Logarithm	$x = \ln p$	$s_x = \frac{s_p}{p}$
Antilogarithm	$x = anti\log_{10} p$	$s_x = 2.303 \cdot x \cdot s_p$
Natural Antilogarithm	$x = e^p$	$s_x = x \cdot s_p$

* p, q, and r are experimental variables whose standard deviations are s_p, s_q, and s_r, respectively: y is a constant.

Least Squares Error Analysis

With a set of data points, the error for a linear fit can be found following these equations: (x and y are the data points and N is the number of data points)

$y = m \cdot x + b$: line equation

$\Delta = N \cdot \sum x^2 - \left(\sum x\right)^2$:

$b = \frac{\sum x^2 \cdot \sum y - \sum x \cdot \sum xy}{\Delta}$: y-intercept

$m = \frac{N \cdot \sum xy - \sum x \cdot \sum y}{\Delta}$: slope

$\sigma_y = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}\left(y_i - b - mx_i\right)^2}$: standard deviation of the y's

$\sigma_b = \sigma_y \sqrt{\frac{\sum x^2}{\Delta}}$: standard deviation of the y-intercept

$\sigma_m = \sigma_y \sqrt{\frac{N}{\Delta}}$: standard deviation of the slope

$y = \left(m \pm \sigma_m\right) \cdot x + \left(b \pm \sigma_b\right)$: line equation with error

This system of equations is what Microsoft Excel uses in its LINEST function.